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WEALTH ACCUMULATION & ECONOMIC PROGRESS

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Abstract

In an evolutionary dynamic economic theory the accumulation of durable goods (i.e. wealth) is a key feature. Here we show that the wealth of individual economic agents can be measured by the progress function. PF is a function of goods and money under straightforward assumptions, notably the \`no-loss' rule for transactions. Explicit formulae for wealth from the PF are derived. We also show how the compatibility of the PF and the neoclassical economics deriving the conventional utility functions from the PF.

Introduction

In our earlier works [Bródy, Martinás, Sajó,85,94] [Martinás 89] [Ayres & Martinás 90,94,95] we showed that the stock representation of the economic agents, together with the "no-loss" rule for transactions are sufficient to guarantee the existence of a non-decreasing function which we called the progress function. Here we show that this function can be interpreted as the wealth of an economic agent. First we discuss the properties of the wealth function, and we show that the PF satisfies the requirements for being a measure of wealth.

The \`no-loss' rule allows for the possibility of optimization at each stage, but does not presume it. If besides the "no loss" rule, economic agents are assumed to be perfectly rational and subject to budget constraints we get a PF utility function. In static case it will be reduced to a neoclassical utility function. The important point is that the PF utility function can be introduced without the transitivity property of the preference ordering.

Properties of a wealth function

In the present paper, for ease of explanation, we first sum up the essential features of a function measuring the wealth. If such a quantity exists, then it has to fulfill the following criteria, and if these criteria are fulfilled, then the quantity can be called wealth function.

(i) Since wealth is a positive attribute (in the absence of the possibility of debt), a function that measures wealth must be non-negative.

(ii) All goods and money that are owned[1] by the economic agent are included within the function.

[1]The terms "own", "owned", "ownership" etc. are used hereafter as shorthand for a more cumbersome phrase, such as "to which the economic agent has enforceable exclusive access". The possible existence of legal encumbrances like mortgages or loans can be ignored for present purposes.

(iii) An increase in the agent's ownership of stocks of beneficial goods or money results, ceteris paribus, in an increase in the agent's wealth.

(iv) An agent's wealth can only increase or stay constant (but never decrease) as a consequence of voluntary acts of exchange.[2]

(v) Wealth is not necessarily additive, but it has the property of homogeneity in the first degree: if the quantity of every stock is multiplied by a factor a, wealth is multiplied by factor a.

Statements (i) and (iii) are consistent with the everyday use of the word. If one or more them is violated, the function cannot measure the wealth.

Statement (ii): Any theoretical accounting framework must recognize the parallel existence of both stocks and flows of commodities (and funds). In a perfect equilibrium state with constant unchanging stocks, all flows must balance and a representation in terms of flows alone can be justified. This is, essentially, the underlying assumption of neo-classical economic theory, and especially, general equilibrium theory. However, in real accounting systems stock changes must be considered in order to balance the accounts during any period. Moreover, a system of exchange with unchanging stocks cannot grow or expand by any endogenous mechanism. This is one reason for the difficulty of reconciling static general equilibrium theory with (non-homothetic) economic growth.

The distinction between goods (property) and services is ignored, for convenience, in most standard treatments, on the argument that a "good" produces a stream of "services".

We insist, however, that the distinction between services purchased – or not – on an "as-needed" (like the telephone) basis are very different in practice from services provided by goods that are owned. The existence of very large transaction costs in real markets makes this distinction vital. For many products there is no resale market at all, and even for a car, the resale market is very restricted; an ordinary buyer who resold the next day would lose at least 20%, if not more, in the turnaround.

In this paper, we show that one feasible starting point for a more general dynamic theory is to introduce the notions of ownership, variable stocks and product life to the conventional theory of demand/consumption. This means that the independent variables, used in characterizing the economic agents will be the stocks (instead of the flows) of goods and money. Since dynamic models of economic growth depend on introducing the notion of capital investment and capital goods – which are stocks – this approach is hardly alien.

Statement (iv) is weak formulation of the maximization principle. It is generally consistent with notions of limited or bounded rationality introduced long ago by Herbert Simon and others.[3] An important caveat is that the no-loss rule can be applied only to voluntary choices.

[2] The payment of taxes (for example) is considered to be involuntary.

[3] The organization-theory challenge was primarily due to Herbert Simon [Simon 55, 59, 65], and by Richard Cyert & James March [Cyert & March 63]. In recent years, moreover, a number of experiments by behavioral psychologists (e.g. [Tversky & Kahneman 74, 81, 87], and also some interesting experiments by economists [Plott 86: Smith 86: Sterman 87, 89], among others) have demonstrated beyond serious doubt that the decision rules actually adopted by most people, including managers, in most situations are not **rational** in the NM sense. (In fact, both Smith's and Sterman's simulations strongly suggest that the most common decision rule can be stated very simply as "follow the leader"). Yet the neo-classical mainstream of economics has not surrendered. On the contrary, the so-called "rational expectations theory" asserts that impersonal markets (such as the stock market) really do optimize, because they are able to overcome the computability limitations of individuals and take rational account of *all* information available at a given time (see, for

Taxation, theft, confiscation, and depreciation due to wear and tear are all examples of non-voluntary processes. Nevertheless the existence of voluntary processes gives rise to a framework (the progress function formalism) in which involuntary processes can later be included.

Statement (v) is only needed to derive an explicit mathematical form for the progress function. In principle, we will show (Appendix) that an alternative approach to the derivation can be based directly on utility theory, as outlined by von Neumann & Morgenstern [von Neumann & Morgenstern 44].

An important caveat is that the no-loss rule can be applied only to voluntary choices. Taxation, theft, confiscation, and depreciation due to wear and tear are all examples of non-voluntary processes. Nevertheless the existence of voluntary processes gives rise to a framework (the progress function formalism) in which involuntary processes can later be included.

Assumptions concerning Economic Decisions, Economic Units (EU's) & Economic Systems needed for the Progress Function

Economic actors perform functions consumption and exchange, and accumulate private tangible goods, both "consumption goods" with short lifetimes and "durable goods" with long lifetimes. We explicitly assert that economic decisions by our actors will be based on preferences for long term accumulation of goods (including durables and money) rather than by preferences for immediate consumption. Translated into immediate decisions to buy, sell, transform or hold, these long-term preferences imply that short-term decisions must be stock-dependent in a much more fundamental sense than is implied by declining marginal utility.

To proceed further we articulate several other assumptions:

1. The existence of an economic unit (EU), which may (or may not) be one of many units which together constitute an economic system (ES). An EU is defined for our purposes as the smallest entity capable of ownership, exchange or consumption.

2. If the EU's are part of an ES then a system-wide medium-of-exchange can be assumed. It is called money.

3. An EU is capable of either consumption, production or exchange. (An EU would normally be either a firm or an individual.) We assume that EU's may interact directly with each other via pairwise exchanges of goods[4] and money.

4. Both material goods and money are conserved in exchange transactions. (Obviously goods are not conserved in consumption).

5. The criterion for a positive decision to exchange one asset (or money) for another is that the EU not be left worse off than it was before.

6. Well-offness (welfare) is a function of the economic state of the EU. The latter is determined by the stocks of money and goods owned, which can be sold,

example [Sargent & Hansen 81]). The argument still rages.

[4] In general, services can also be exchanged for goods (or other services). However we restrict ourselves at this stage to transactions involving only tangible goods or labor. The extension to other services will be considered later.

(exchanged) or consumed.

7. For purposes of this paper, material goods do not depreciate. (This assumption is fairly easy to relax, as we hope to show in a subsequent paper).

It is convenient, and important, for what follows to distinguish between extensive variables, which measure the size or magnitude of the system, and intensive variables, which measure characteristics that are independent of size. Stocks of goods and money are examples of extensive variables. Intensive variables are usually ratios of extensive variables. The time rate of change of extensive variables can always be expressed in terms of flows (inputs and outputs) and sources or sinks[5]:

$$\frac{dX_i^\alpha}{dt} = J_i^\alpha + S_i^\alpha \quad (1)$$

where J_i^α is net imports of the ith commodity (imports minus exports) and S_i^α is the net production (production minus consumption) of the ith commodity within the α^{th} EU. By similar logic, one can write

$$\frac{dM^\alpha}{dt} = -\sum_i P_i^\alpha J_i^\alpha + I^\alpha \quad (2)$$

where P_i^α is the money cost (price) of one unit of the ith good or commodity and I^α is the net financial inflow, i.e. the difference between credits, subsidies, interest or dividends received, loans or investments from outside the EU (e.g. dividends or capital gains) and debits (interest or dividends paid, taxes paid, losses on external investments, etc.).

The above assumptions are well known in the economics literature. The exception is that we do not assume the maximizing behavior. The no loss rule for decisions allows also the maximization, but it does not require it. The other is that here we explicitly require the presence of the stocks. One of the consequences is our decentralized version of the exchange process. In our case it is not necessary to assume an equilibrium market price known to all EU's. In other words, the money cost (price) can change from transaction to transaction.

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[5] The general bookkeeping equation for any extensive quantity *X* (such as a physical commodity) is

$$\frac{dX}{dt} = F + G$$

where *F* is a generalized *current* (inflow) that crosses the boundary of the economic unit and *G* is a generalized *source* (or, with a negative sign, a sink). By assumption *X* can be any commodity that can be bought, sold, produced or consumed, including money or shares of stock. A quantity is conserved if G = 0. This is the so-called "materials-balance" condition.

Properties of the Progress Function *Z*

In our previous papers [Bródy, Martinás, Sajó 85,94], [Martinás 89], [Ayres@ Martinás 91,94] it was shown that the listed assumptions ensure the existence of a unique function of assets and money for each actor. We called this the progress function (PF). This function has the property that it never decreases in any voluntary exchange transaction. The situation can be restated: a necessary criterion for every voluntary transaction between two parties is that the progress functions must not decrease for either party. (For simplicity, we called this the "no-loss" rule).

The non-decreasing progress function has no counterpart in standard static theory, where there is no time dimension at all. In the dynamic case, however, the idea of increasing or decreasing some stock is usually explicit. The usual problem is to allocate production or consumption optimally over time. For instance, most of the examples in the economics literature have assumed an initially fixed resource base that is used up (consumed) over some time horizon. The case we now want to consider is essentially an elaboration of the Ramsey case: accumulation of tangible wealth (durable goods and money) over time. In principle, the dynamic optimization machinery is applicable here, at least, in some simplified models. However, our computational procedure is rather different.

The progress function Z is a function of extensive variables because of assumption (6). All goods that are owned by the economic agent are included within the function together with the EU's stock of money. These stocks are the only internal variables. In mathematical form:

$$Z = Z(X_1, X_2, \ldots, M) = Z(\overline{X}, M) \tag{3}$$

Z has no explicit time-dependence. Time-dependence occurs only through changes in the stocks held by the EU, viz.

$$\frac{dZ}{dt} = \sum_i \frac{\partial Z}{\partial X_i} \frac{dX_i}{dt} + \frac{\partial Z}{\partial M}\frac{dM}{dt} = \frac{1}{T}\left[\sum_i w_i \frac{dX_i}{dt} + \frac{dM}{dt}\right] \tag{4}$$

For convenience, the following short-hand notation can be introduced:

$$\frac{1}{T} = \frac{\partial Z}{\partial M} \tag{5}$$

and

$$w_i = T\frac{\partial Z}{\partial X_i} \tag{6}$$

For beneficial goods the functions w_i must be positive. The same is true of T. The no-loss behavior of EU's implies restrictions on voluntary exchanges, viz. a necessary condition for every voluntary action by an EU is that its progress function must not decrease in consequence.

To clarify the economic meaning of w_i, consider an exchange process. For the sake of simplicity, assume a small increment of one good (the i[th]) is sold $(dX_i < 0)$ or bought $(dX_i > 0)$, for an increment of money dM. The EU's marginal change of wealth will be:

$$dZ = \frac{1}{T}\left(w_i dX_i + dM \right) \quad \textbf{(7)}$$

Let us now define the price p_i for this (private) transaction as follows:

$$p_i = - \frac{dM}{dX_i} \quad \textbf{(8)}$$

The incremental change in the EU's progress function Z becomes

$$dZ = \frac{1}{T} (w_i - p_i) dX_i \quad \textbf{(9)}$$

Now w_i can be interpreted as the internal value of one unit of the i[th] good to the economic unit, expressed in money terms. The economic unit feels itself richer by an amount proportional to (w-p)/T, if it sells or buys the good at the price p. In general

$$dZ = \sum_{i=1}^{n} \frac{w_i}{T} dX_i + \frac{dM}{T} \quad \textbf{(10)}$$

The no-loss property of the Z function implies that for every voluntary economic action $dZ \geq 0$. Thus, $p_i > w_i$ is a condition such that the EU is willing to sell. If the price is higher than the value to the EU, then his/her willingness to buy diminishes. Similarly, $p_i < w_i$ ensures that the EU does not sell the good. This property of Z leads us toward a new quantitative measurement of value. In principle, there exists a limiting price p_o at which the EU is indifferent between buying, holding or selling. (This price is sometimes called the reservation price.) Nevertheless, this limiting price can be accessed from non-equilibrium measurements. The new measure does not apply only to market equilibrium: It is applicable in any pairwise encounter between economic units.

Imagine a collection of N EU's, indexed by i = 1, ... N. Each is endowed with a different stock of goods and money, and a different set of long-term preferences. Select the n[th] EU, namely EU_n. It by assumption, unique and identifiable. Every EU offers to trade with others, one at a time, in some random sequence. The EU with the lowest index number initiates the process (i.e. makes an offer to buy, or sell, a certain quantity of some commodity at some price. The offer may or may not be accepted. The collection of EU's constitute an ES. Now imagine a large number of replicas of the ES, differing only in terms of the sequence of interactions. If one could conduct such a gedanken experiment, monitor the results, and plot the results of many offers might look like Figure 1. In the absence of the no-loss rule, the results might look like Figure 2.

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Figure 1: Exchanges with the no-loss property

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Figure 2: Exchange without the no-loss property

In Figure 1, there is a maximum offer price p_o at which the EU will agree to buy, and a minimum price p_s at which the EU will sell. The inequality $p_s < p_o$ holds. As the number of increases, the difference $p_s - p_o \to 0.$ We can now formalize a definition of internal value: For the EU the internal value of the i^{th} good is p_o.

Repeating the above experiment with other cases where the EU begins with differing stocks, the result will be to determine value as a function of the extensive variables, viz.

$$w_i = w_i(\overline{X}, M) \tag{11}$$

In summary, the internal value of a good is defined for (and by) each EU. Moreover, it is known only to the EU. It is important to note that, since each EU may have a different decision rule, two EU's would not be likely to assign the same value for each good, even if both were momentarily in the same economic state.

The Liquidity T

In addition to the progress function and the value-functions a further new variable was introduces above T. In effect 1/T measures the internal value of money to the EU, in the same way that w measures the value of a good. If the EU is in a state of T_1, and it gains an increment dM of money, then its wealth increases marginally as

$$dZ = \frac{dM}{T_1} \tag{12}$$

Now we can compare two economic states 1,2. Assuming a given marginal increase in wealth dZ

$$\frac{T_1}{T_2} = \frac{dM_1}{dM_2} \tag{13}$$

Thus T_1/T_2 is the ratio of money increments needed to attain the same increment of wealth increase for the economic unit in states 1 and 2, respectively. This variable cannot be determined on the basis of simple exchanges, as it does not appear there. Nevertheless T is defined by the value functions, over their whole range. The symmetry of the second partial derivatives of the progress function implies that the T function cannot be chosen arbitrarily, but has to obey the following relations:

$$\frac{\partial^2 Z}{\partial X_i \partial X_k} = \frac{\partial^2 Z}{\partial X_k \partial X_i} \tag{14}$$

Since $\partial Z/\partial X_i = w_i/T$ and $\partial Z/\partial M = 1/T$, we obtain:

$$\frac{\partial \ln T}{\partial X_i} - w_i \frac{\partial \ln T}{\partial M} = - \frac{\partial w}{\partial M} i \tag{15}$$

In (15) it is permissible to replace Z with an arbitrary function $Z^*(\ ) = f(Z(\ ))$. The relationships above define the T function except for a multiplying factor df(Z)/dZ. To eliminate this ambivalence we need a further restriction. The linear homogeneity property of the progress function ensures that Z is a sum of bilinear products of stocks and their values. The homogeneity property means that

$$Z = \sum_i \frac{w}{T} i X_i + \frac{M}{T} \tag{16}$$

Differentiating both sides by m, and taking m = 1 one obtains

$$Z = \sum_i \frac{\partial Z(mX, \cdots, mM)}{\partial X_i} \frac{d(mX_i)}{dm} = \sum_i \frac{\partial Z(X, \cdots, M)}{\partial X_i} X_i . \tag{17}$$

As by definition,

$$dZ = \sum_i \left(\frac{w_i}{T}\right) dX_i + \frac{dM}{T}, \tag{18}$$

and from the bilinear form follows, that

$$dZ = \sum_i \left(\frac{w_i}{T}\right) dX_i + \left(\frac{1}{T}\right) dM + \sum_i X_i d\left(\frac{w_i}{T}\right) + Md\left(\frac{1}{T}\right) \tag{19}$$

so

$$\sum_i X_i d\left(\frac{w_i}{T}\right) + Md\left(\frac{1}{T}\right) = 0. \tag{20}$$

Thus, we obtain the following differential equation for T:

$$d\left(\frac{1}{T}\right) = \frac{\sum_i X_i dw_i}{\left[\sum_i w_i X_i + M\right]} \tag{21}$$

which can be integrated from state 1 (X_1,...M) to state 2 ((X'_1,...M')

$$\left(\ln \frac{T_2}{T_1}\right) = \int_1^2 \frac{\sum_i X_i dw_i}{\sum_i w_i X_i + M} \tag{22}$$

yielding

$$T = T_o \exp\left[\frac{\int \sum_i X_i dw_i}{\sum_i w_i X_i + M}\right] \tag{23}$$

We are free to choose T_o arbitrarily. An alternative derivation that does not require the homogeneity assumption is given in the Appendix.

The progress function itself is now fully specified. If (by series of transactions) the EU evolves from the state a (X^a_1,...M^a) to the state b (X^b_1,...M^b) its progress function will change as follows:

$$\Delta Z = \int_a^b \sum_i \frac{w}{T} i \, dX_i + \frac{dM}{T} \tag{24}$$

Clearly the progress function is a cardinal quantity. It is uniquely defined by the economic characteristics of the economic unit. We have only to specify its unit of measurement and to define its "zero point" or reference point.

A `Simple' Progress Function

Functions Z and T are uniquely characteristic of each EU. An explicit determination can only be derived from experience. Nevertheless the present formalism makes it possible to guess the mathematical form of these functions from general considerations. One of the simplest expressions for T satisfying all the required conditions is:

$$T = \frac{M}{\sum_i g_i X_i} \tag{25}$$

where the g_i are constant coefficients. Putting this form of T into equation (22) one obtains

$$\sum_i \left(g_i dX_i - \frac{dM}{M} g_i X_i \right) = \sum_i X_i d\frac{w_i}{T} \tag{26}$$

Simple mathematics yields explicit expressions for the internal value functions:

$$V_i = \left[\frac{g_i M}{\sum_i g_i X_i}\right]\left[\ln\left(\frac{M}{X_i}\right) + c_i\right] \tag{27}$$

where c_i is a constant of integration. The progress function has a logarithmic dependence on stocks:

$$Z = \sum_i g_i X_i \left[\ln\left(\frac{M}{X_i} \right) + c_i \right] = \sum_i g_i X_i \ln\left(\frac{M}{k_i X_i} \right) \qquad \textbf{(28)}$$

An important caveat must be emphasized: the above is only one possible form. In fact, there is no guarantee that this particular form is the correct one in any given case. Nor is it necessarily true (or even likely) that all EU's in the real world will be characterized by a T-function (or the corresponding V-, Z-functions) having the same form. The actual form would have to be determined by experiment or observation on a case-by-case basis.

The relation of the Wealth and the Progress Function

The requirement for the wealth function are satisfied by the Progress function, namely

(i) Since wealth is a positive attribute (in the absence of the possibility of debt), a function that measures wealth must be non-negative. - Z is positive

(ii) All goods and money that are owned[6] by the economic agent are included within the function. $Z = Z(X_1,\ldots M)$

(iii) An increase in the agent's ownership of stocks of beneficial goods or money results, ceteris paribus, in an increase in the agent's wealth.
- for beneficial goods $w > 0$, so $dZ > 0$.

(iv) An agent's wealth can only increase or stay constant (but never decrease) as a consequence of voluntary acts of exchange.[7]
Assumption 6 ensures it.

(v) Wealth is not necessarily additive, but it has the property of homogeneity in the first degree.

The progress function gives a new approach to the measurement of wealth.

Dynamics, Utility & Non-equilibrium Economic Processes

The economic system consist of economic units (EU). A market, an economy can be considered as a collection of economic units. The dynamics of the system is characterized if all unit is known. This description starts from the EU's so the equilibrium state of the system is not needed, for calculations, but we can investigate the processes leading to equilibrium.

The progress function itself characterizes only the state of EU. In voluntary processes the no-loss rule is reflected in the non-decreasing character of the progress function. As we have

[6] The terms "own", "owned", "ownership" etc. are used hereafter as shorthand for a more cumbersome phrase, such as "to which the economic agent has enforceable exclusive access". The possible existence of legal encumbrances like mortgages or loans can be ignored for present purposes.

[7] The payment of taxes (for example) is considered to be involuntary.

noted repeatedly, the no-loss rule excludes exchanges requiring coercion. For dynamics we need an extra assumption for the choice from the permitted possibilities. The no-loss rule does not determine when or under what conditions a trade will occur. It only defines the conditions under which no trade can occur. It does not require maximizing behavior, but in general it does not exclude it, except in a few special cases (such as lotteries, see below). It therefore implies far less omniscience on the part of economic agents than "perfect" rationality.

Here we show that the progress function approach is compatible with the neoclassical economics. When for the dynamics we introduce the maximization principle we get back the utility maximization principle.

Let us now revisit the neoclassical model. The neoclassical consumer, who has perfect knowledge on the market, faces a problem that can be formulated as follows [Kreps 90]:

> "Choose the consumption bundle x that is best according to preferences, subject to the constraint that total cost of x is no greater than the consumer's income."

In the progress function approach the corresponding consumer's problem would be:

Assumption 8:

> "The EU chooses the consumption bundle x that gives the highest increase in wealth, subject to the constraint that total cost of x is no greater than the EU's income."

We have to find the vector of q, of goods and money which maximizes his/her progress function. (Bear in mind that, normally, we do not assume maximizing behavior. Assumption 9 is possible but not necessary addition). The quantity to be maximized is similar to the utility function. However to stress the difference we refer to Z-utility and use the subscript Z, viz.

$$U_Z\,(\overline{X}, M, \overline{p}, \overline{x},\,) = Z\,(\overline{X}+\overline{x}, M-\overline{p}\overline{x}) - Z\,(\overline{X}, M) \qquad \textbf{(29)}$$

If $\overline{X}$, $\overline{p}$, M are constant then U_Z reduces to

$$U_Z = U_Z(\overline{x}) \qquad \textbf{(30)}$$

The indirect utility function can now be introduced. This function defines how much utility the consumer receives from his/her optimal choice(s) at prices p and income Y. The function v_Z measures the wealth gain of the consumer (EU) resulting from its optimal choice(s), viz.

$$v_Z\,(\overline{p}, Y) = \max\{\,U_Z(\overline{x}) : \overline{p}\overline{x} \le Y \text{ and } x \ge 0\,\} \qquad \textbf{(31)}$$

The marginal Z-utility derived from the U_Z function is:

$$\frac{\partial U}{\partial x_i} = w_i - p_i \qquad (32)$$

The marginal Z-utility of the i[th] good is the wealth increment of the EU that results from exchanging a unit of its money for a unit of this good.

The Z-utility function has most features of conventional utility functions, nevertheless there are some differences. Most important, the conventional utility function U can be rescaled. It has no particular cardinal significance. It is legitimate to replace U with any monotonous function $f(U(\ ))$. By contrast, U_Z is fully specified by the characteristics of the economic unit. There is only one free parameter in Z, namely T_o, the unit in which wealth (utility) is measured. Similarly, the conventional value function v is homogeneous of degree zero in prices p and income Y. By comparison, v_Z is homogeneous of degree zero in p and Y only in the special case of monetary inflation, where the general money stock in the economic system is increased, i.e. there is a rescaling of money. In all other cases, the Z-values of the EU can change in a non-homogeneous manner by the prices.

If we restrict our economic system interactions to pair-wise exchanges, then the wealth maximization principle defined the pair-wise exchanges, and the production decisions too. It allows to investigate non-equilibrium economic systems. When stock changes are taken seriously as an important phenomenon of a dynamic economic system the important features of a static general equilibrium system (e.g. market-clearing price formation) can be retained. More important, the dynamic system incorporates a new feature not present in the static general equilibrium system: the possibility of accumulation (or depletion) of private stocks of tangible goods and money.We published some results of our computer simulations based this local wealth maximization criteria.[Ayres & Martinás-a 1990].

Summary

Here we have shown that starting from economic agents with no-loss behavior, a new type of economic function, called progress function can be introduced. This gives a measure of the wealth of the agents, so its time behavior can be described by the change of its wealth. On the observed economic behavior one can guess the form of PF. Here some simplest forms were shown.

The present approach gives a new way to investigate economic systems, as a collection of economic units. If we add, as a further restrictions the maximizing behavior then the neoclassical utility function can be derived from the progress function. The present approach is a generalization of the neoclassical economics. It allows the investigation of non-equilibrium systems, and their time evolution.

Appendix: An Alternative Approach to Construction of the *Z*-function

The standard von-Neumann-Morgenstern expected utility paradigm applied to our model starts with an imaginary situation where individual economic agents are offered choices involving different bundles of goods and money, with different probabilities and must choose among them to establish indifference points. Suppose an EU with two such offers, namely M_1 money with p_1 probability, or M_2 money with p_2 probability. Since there is no cost involved, the no-loss rule does not apply. The EU is indifferent between the two possibilities when the wealth probability products are equal i.e.

$$p_1 \left(Z(M_o+M_1) - Z(M_o) \right) > p_2 \left(Z(M_o+M_2) - Z(M_o) \right) \quad (33)$$

This can be solved for Z:

$$Z(M_o+m) = \frac{p(m)}{p(m_o)}\left[Z(M_o+m_o) - Z(M_o)\right] + Z(M_o) \quad (34)$$

The liquidity T is the derivative of Z with respect to m:

$$\frac{1}{T} = \frac{\partial Z(M_o+m)}{\partial m} = \frac{\partial p(m)}{\partial m}\frac{1}{p(m_o)}\left[Z(M_o+m_o) - Z(M_o)\right] \quad (35)$$

This last equation defines T as a function of money. The constant multiplier signifies the arbitrariness of the choice of units of wealth, and correspondingly, of liquidity.

In the case of composite choices one has to be cautious. The economic behavior of real economic agents suggests the no loss rule. The above scheme makes it possible to map the wealth function over all combinations of arguments by construction of Z = constant surfaces.

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References

[Ayres & Martinás-a 1990] Ayres, Robert U. & Katalin Martinás, Self-Organization of Markets & the Approach to Equilibrium, Working Paper (WP-90-18), International Institute for Applied Systems Analysis, Laxenburg Austria, 1990.

[Ayres & Martinás-b 1990] Ayres, Robert U. & Katalin Martinás, A Computable Economic Progress Function, Working Paper (WP-90-18), International Institute for Applied Systems Analysis, Laxenburg Austria, April 1990. in: Robert U. Ayres, Information, Evolution and Economics, American Institute of Physics, 1994. Chapter.7.2.

[Ayres & Martinás-b 1990] Ayres, Robert U. & Katalin Martinás, A Non-Equilibrium Evolutionary Economic Theory, Working Paper (WP-90-25), IIASA, Laxenburg Austria, 1990. in: Thermodynamics and Economics, Ed. Burley, Kluwer, 1994.

[Bródy 1986] Bródy, András, Katalin Martinás, Konstantin Sajó, Essay on Macroeconomics, Acta Oec. 36, December, 1985 :305 in: Thermodynamics and Economics, Ed. Burley, Kluwer, 1994.

[Cyert & March 1963] Cyert, Richard & James March, A Behavioral Theory of the Firm, Prentice-Hall, Englewood Cliffs NJ, 1963.

[Kreps 1990] Kreps, David M., A Course in Microeconomic Theory, Harvester Wheatsheaf, New York, 1990.

[Martinás 1989] Martinás, Katalin, About Irreversibility in Microeconomics, Research Report (AHFT-89-1), Department of Low Temperature Physics, Roland Eotvos University, Budapest, March 1989.

[Plott 1986] Plott, C.R., ``Laboratory Experiments in Economics: The Implications of Posted Price Institutions'', Science **232**, May 9, 1986 :732-738.

[Ramsey 1942] Ramsey, F.P., ``A Mathematical Theory of Saving", Economics Journal **38**(152), December 1942 :543-559.

[Sargent & Hansen 1981] Sargent, T.J. & L.P. Hansen. ``Linear Rational Expectations Models of Dynamically Interrelated Variables", in Rational Expectations & Econometric Practice, University of Minnesota Press, Minneapolis MN, 1981.

[Simon 1955] Simon, Herbert A., ``A Behavioral Model of Rational Choice", Quarterly Journal of Economics **69**, 1955 :99-118. [Reprinted in Models of Man, Wiley, NY, 1957]

[Simon 1959] Simon, Herbert A., ``Theories of Decision-Making in Economics", American Economic Review **49**, 1959 :253-283.

[Simon 1965] Simon, Herbert A., Administrative Behavior, Free Press, New York, 1965. 2nd edition.

[Smith 1986] Smith, V.L., ``Experimental Methods in the Political Economy of Exchange", Science **234**, October 10 1986 :167-173.

[Sterman 1987] Sterman, John D., ``Testing Behavioral Simulation Models by Direct Experiment", Management Science **33**, 1987 :1572-1592.

[Sterman 1989] Sterman, John D., Deterministic Chaos in an Experimental Economic System, International Symposium on Evolutionary Dynamics & Non-Linear Economics, University of Texas, Austin TX, April 1989.

[Tversky & Kahneman 1974] Tversky, A. & D. Kahneman, ``Judgment Under Uncertainty: Heuristics & Biases", Science **185**, September 27, 1974 :1124-1131.

[Tversky & Kahneman 1981] Tversky, A. & D. Kahneman, ``The Framing of Decisions & the Psychology of Choice", Science **211**, January 30, 1981 :453-458.

[Tversky & Kahneman 1987] Tversky, A. & D. Kahneman. ``Rational Choice & the Framing of Decisions", in: Hogarth & Reder(eds), Rational Choice: The Contrast Between Economics & Psychology, University of Chicago Press, Chicago, 1987.

[von Neumann & Morgenstern 1944] von Neumann, John & Oskar Morgenstern, Theory of Games & Economic Behavior, Princeton University Press, Princeton NJ, 1944.